

Factors controlling the time-delay between peak CO₂ emissions and concentrations

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Abstract

Carbon-dioxide (CO_2) is the main contributor to anthropogenic global warming, and the timing of its peak concentration in the atmosphere is likely to govern the timing of maximum radiative forcing. It is well-known that dynamics of atmospheric CO_2 is governed by multiple time-constants, and here we approximate the solutions to a linear model of atmospheric CO_2 dynamics with four time-constants to identify factors governing the time-delay between peaks in CO_2 emissions and concentrations, and therefore the timing of the concentration peak. The main factor affecting this time-delay is the ratio of the rate of change of emissions during its increasing and decreasing phases. If this ratio is large in magnitude then the time-delay between peak emissions and concentrations is large. Therefore it is important to limit the magnitude of this ratio through mitigation, in order to achieve an early peak in CO_2 concentrations. This can be achieved with an early global emissions peak, combined with rapid decarbonization of economic activity, because the delay between peak emissions and concentrations is affected by the time-scale with which decarbonization occurs. Of course, for limiting the magnitude of peak concentrations it is also important to limit the magnitude of emissions throughout its trajectory, but that aspect has been studied elsewhere and is not examined here. The carbon cycle parameters affecting the timing of the concentration peak are primarily the long multi-century time-constant of atmospheric CO_2 , and the ratio of contributions to the impulse response function of atmospheric CO_2 from the infinite time-constant and the long time-constant respectively. Reducing uncertainties in these parameters can reduce uncertainty in forecasts of the radiative forcing peak.

Keywords

Global climate change; carbon dioxide; peak radiative forcing; climate change mitigation; decarbonization.

1 Introduction

As countries agree on commitments towards a new international climate treaty to be decided in 2015 (*UNFCCC* (2014a,b)), these will include mitigation of not only carbon-dioxide (CO_2) but also other climate forcers (*UNFCCC* (2014c)). CO_2 is, and is likely to remain, the largest contribution to radiative forcing (*Forster et al.* (2007); *Myhre et al.* (2013)). Limiting long-term warming requires limiting the growth in global CO_2 emissions, and eventually reducing these emissions. If the present increasing trend in global CO_2 emissions is eventually reversed so that an emissions peak occurs, the corresponding peak in concentration will be delayed because of its long atmospheric lifetime (*Allen et al.* (2009); *Meinshausen et al.* (2009);

Mignone et al. (2008)). A CO_2 concentration peak would be a significant event for global climate: it would govern the maximum contribution of CO_2 emissions to radiative forcing. Furthermore, assuming that CO_2 continues to be the major contribution to radiative forcing, then its peak concentration will strongly influence the magnitude and timing of peak global warming.

The Earth’s CO_2 cycle is complex, involving multiple reservoirs that maintain exchanges occurring at very different rates (*Archer et al.* (1997); *Cox et al.* (2000); *Falkowski et al.* (2000)). The most rapid uptake of excess CO_2 is by the surface ocean and land biosphere (*Pierrehumbert* (2014)). Progressively slower processes involve mixing with the deep-ocean, reduction of ocean acidity due to dissolution of carbonates, and uptake of excess atmospheric CO_2 via reaction with CaCO_3 or silicate rocks on land (*Archer et al.* (1997); *Archer and Brovkin* (2008); *Archer et al.* (2009)). The last two processes require many tens of thousands of years so that, on the timescales of the next few centuries, their contributions can be effectively neglected. Equivalently their effects can be treated as occurring with infinite time-constant. Accurate characterization of the different processes involved, in order to describe the fate of excess atmospheric CO_2 , requires coupled climate-carbon-cycle or Earth-system models; such models have been employed to describe effects of mitigation scenarios on CO_2 in the atmosphere (*Petoukhov et al.* (2005); *Friedlingstein et al.* (2006)). As the mitigation of CO_2 emissions unfolds, these and similar models will play important roles in estimating the consequences for atmospheric CO_2 , including the timing and magnitude of its peak concentration.

This paper solves a linear model of atmospheric CO_2 with four time-constants (*Joos et al.* (2013)) to understand the factors controlling the time-delay between peaks in emission and concentration and therefore the timing of the concentration peak. Previous studies have described the relationship between mitigation and warming, and highlighted the importance of rapid mitigation (for e.g. *Socolow and Lam* (2007); *Allen et al.* (2009); *Allen and Stocker* (2014); *Huntingford et al.* (2012)). Here we focus specifically on solving the model of atmospheric CO_2 analytically, to identify some of the important factors controlling the time to the concentration peak of CO_2 .

2 Models of emissions and carbon cycle

2.1 Carbon cycle model

Joos et al. (2013) estimated the impulse response of CO_2 in the Earth’s atmosphere, by averaging a group

of Earth System Models. They estimated a mean response with four time-constants

$$h(t) = 0.276e^{-t/4.30} + 0.282e^{-t/36.5} + 0.224e^{-t/394} + 0.217 \quad (1)$$

which we apply, and compute atmospheric concentration using

$$u(t) = \int_0^t m(z) h(t-z) dz + u_P \quad (2)$$

where $m(z)$ is anthropogenic emissions in concentration units, i.e. the rate of increase in concentration in the hypothetical case of infinite atmospheric lifetime. The constant term u_P is preindustrial concentration. Concentration is noted in parts per million (ppm). Atmospheric emission of CO_2 in the year 2013 was 36×10^{12} kg, equivalent to 4.5 ppm.¹ We furthermore describe the impulse response function symbolically as

$$h(t) = \sum_{i=1}^K \mu_i e^{-t/\tau_i} \quad (3)$$

with $\sum_{i=1}^K \mu_i = 1$ and $\{\tau_1, \tau_2, \tau_3, \tau_4\} = \{4.4, 36.5, 394, \infty\}$ following the results of *Joos et al.* (2013). The infinite time-constant approximates the effects of very slow processes involving buffering of ocean acidity by dissolution of carbonates and the uptake of CO_2 in the weathering of rocks.

There is uncertainty in the function $h(t)$, with different earth system models likely to yield different results (*Joos et al.* (2013)). While the present paper does not characterize this uncertainty, Section 3.3 considers the influence of changing those parameters in this 4-time-constant model that are shown to affect the timing of the concentration peak.

2.2 Emissions model

The model of emissions $m(t)$ is very simple, and chosen so as to describe emissions using a few different parameters that can be readily interpreted. The emissions model is $m(t) = m_0 (1 + r)^{(\min(t, t_g) - t_0)} e^{-(t - t_0)/\tau_m}$ (*Seshadri* (2015a,b)), with m_0 being present emissions, r the growth rate of gross global product (GDP), and t_0 denoting the present time. It is assumed, for the purposes of the emissions model used here, that GDP increases for t_g years from the present at constant rate r , after which it remains fixed. The term $e^{-(t - t_0)/\tau_m}$ describes the effect of decrease in emissions intensity of economic output, with $\tau_m \rightarrow \infty$ corresponding to the absence of any mitigation, and smaller values of τ_m corresponding to rapid mitigation. This model of emissions has been borrowed from previous work (*Seshadri* (2015a,b)).

¹If CO_2 had infinite lifetime, emissions in the year 2013 would have increased atmospheric concentration by 4.5 ppm.

3 Results

3.1 Peak atmospheric concentration of CO2

With emissions $m(t)$ the atmospheric concentration is $u(t) = u_P + \int_0^t m(z) h(t-z) dz$, and differentiating this equation we obtain for the rate of change of concentration that $u'(t) = m(t) + \int_0^t m(z) \frac{\partial h(t-z)}{\partial t} dz$, where we have used the fact that $h(0) = 1$ from equation (1). Using equation (3) we obtain that $u'(t) = m(t) - \sum_{i=1}^K \frac{\mu_i}{\tau_i} e^{-t/\tau_i} \int_0^t e^{z/\tau_i} m(z) dz$. The last integral can be evaluated by parts to give $\int_0^t e^{z/\tau_i} m(z) dz = \tau_i m(t) e^{t/\tau_i} - \tau_i \int_0^t e^{z/\tau_i} m'(z) dz$, and substituting this yields

$$u'(t) = \sum_{i=1}^K \mu_i e^{-t/\tau_i} \int_0^t e^{z/\tau_i} m'(z) dz \quad (4)$$

for the rate of change of concentration. This result has been derived previously in *Seshadri* (2015c).

For short time-constants $\tau_1 = 4.4$ years and $\tau_2 = 36.5$ years for which $t \gg \tau_i$, $i = 1, 2$, we can approximate the integral $\int_0^t e^{z/\tau_i} m'(z) dz$ by $\tau_i e^{t/\tau_i} m'(t)$. For the very long time-constant that is represented by $\tau_4 = \infty$ the integral becomes $\int_0^t e^{z/\tau_i} m'(z) dz = \int_0^t m'(z) dz = m(t)$. Hence the rate of change of concentration becomes approximately

$$u'(t) \cong (\mu_1 \tau_1 + \mu_2 \tau_2) m'(t) + \mu_3 e^{-t/\tau_3} \int_0^t e^{z/\tau_3} m'(z) dz + \mu_4 m(t) \quad (5)$$

We denote the time to the emissions peak as t_1 , the time to the concentration peak as t_2 , and the time-delay as $\delta t \equiv t_2 - t_1$. Then the integral in equation (5) above can be described as the sum of integrals from 0 to t_1 , and from t_1 to t_2 . Furthermore, defining weighted rates of change of emissions

$$m'_{av,i} = \frac{\int_0^{t_1} e^{z/\tau_3} m'(z) dz}{\int_0^{t_1} e^{z/\tau_3} dz} \quad (6)$$

and

$$m'_{av,d} = \frac{\int_{t_1}^{t_2} e^{z/\tau_3} m'(z) dz}{\int_{t_1}^{t_2} e^{z/\tau_3} dz} \quad (7)$$

we obtain the formula for the rate of change of concentration at time $t = t_2$

$$u'(t_2) \cong (\mu_1 \tau_1 + \mu_2 \tau_2) m'(t_2) + \mu_3 \tau_3 m'_{av,i} e^{-t_2/\tau_3} (e^{t_1/\tau_3} - 1) + \mu_3 \tau_3 m'_{av,d} e^{-t_2/\tau_3} (e^{t_2/\tau_3} - e^{t_1/\tau_3}) + \mu_4 m(t_2) \quad (8)$$

Before proceeding we identify the factors influencing whether the concentration will reach a peak value and

eventually decline, as opposed to continuing to increase to an asymptotic maximum. Consider emissions scenarios where the emissions peaks and then declines to zero so that, eventually, $m'(t) = 0$ and $m(t) = 0$. In that case only the large but finite time-constant $\tau_3 = 394$ years in the model of *Joos et al.* (2013) plays a role in the sign of $u'(t)$. The value of this quantity then becomes, for $t > t_1$, approximately

$$u'(t) \cong \mu_3 \tau_3 m'_{av,i} e^{-t/\tau_3} (e^{t_1/\tau_3} - 1) + \mu_3 \tau_3 m'_{av,d} e^{-t/\tau_3} (e^{t/\tau_3} - e^{t_1/\tau_3}) \quad (9)$$

which is negative if

$$-m'_{av,d} > m'_{av,i} \frac{e^{t_1/\tau_3} - 1}{e^{t/\tau_3} - e^{t_1/\tau_3}} \quad (10)$$

requiring the rate of decrease of emissions to be sufficiently large in magnitude. The larger the average rate of increase in emissions, and the longer the increase persists, the more stringent is the condition on the subsequent decrease in order for concentrations to eventually decrease. Therefore an early peak in global emissions can help stabilize atmospheric CO₂ concentrations, as would be expected.

We now seek an expression for the time t_2 to the CO₂ concentration peak, assuming that mitigation occurs rapidly enough for one to occur. The condition for such a peak is that the rate of change of concentration vanishes. Hence, from equation (8) above

$$u'(t_2) \cong (\mu_1 \tau_1 + \mu_2 \tau_2) m'(t_2) + \mu_3 \tau_3 m'_{av,i} e^{-t_2/\tau_3} (e^{t_1/\tau_3} - 1) + \mu_3 \tau_3 m'_{av,d} e^{-t_2/\tau_3} (e^{t_2/\tau_3} - e^{t_1/\tau_3}) + \mu_4 m(t_2) = 0 \quad (11)$$

and, writing $m(t_2) = m'_i t_1 + m'_d (t_2 - t_1)$, where m'_i and m'_d are average rates of change during emission's increasing and decreasing phases respectively, and collecting terms involving t_2 yields

$$\mu_3 \tau_3 e^{-t_2/\tau_3} \left((m'_{av,i} - m'_{av,d}) e^{t_1/\tau_3} - m'_{av,i} \right) + \mu_4 m'_d t_2 \cong -\mu_3 \tau_3 m'_{av,d} - \mu_4 (m'_i - m'_d) t_1 - (\mu_1 \tau_1 + \mu_2 \tau_2) m'(t_2) \quad (12)$$

We can neglect the last term on the right side of equation (12), since $m'(t_2)$ is comparable in magnitude to $m'_{av,d}$, as will be shown, but $\mu_1 \tau_1 + \mu_2 \tau_2 \ll \mu_3 \tau_3$. Hence the time t_2 to the concentration peak in the model is governed by approximate equality

$$\mu_3 \tau_3 e^{-t_2/\tau_3} \left(m'_{av,i} - (m'_{av,i} - m'_{av,d}) e^{t_1/\tau_3} \right) - \mu_4 m'_d t_2 \cong \mu_3 \tau_3 m'_{av,d} + \mu_4 (m'_i - m'_d) t_1 \quad (13)$$

The approximate equality in equation (13) has been verified in Figure 1c, for emissions scenarios shown in Figure 1a and corresponding concentration graphs plotted in Figure 1b. It is therefore confirmed that the short time-constants can be approximately neglected while studying the concentration peak. The above

equation has to be solved numerically because neither the exponential nor the linear term in t_2 on the left side of the expression can be neglected. That this is the case is shown in Figure 1d, which plots these two terms. Neither term is dominant.

However in order to understand qualitatively the factors influencing the time t_2 , let us imagine that t_2 is so large that the second term in the left side of equation (13), $-\mu_4 m'_d t_2$, were dominant. Then the solution would be given by

$$t_2 \cong \tau_3 \left(\left(\frac{m'_i}{-m'_d} + 1 \right) \frac{t_1}{\tau_3} + \frac{\mu_3}{\mu_4} \frac{m'_{av,d}}{-m'_d} \right) \quad (14)$$

The time to the concentration peak increases with the rate of increase of emissions during their growing phase. It decreases with the rate of decrease of emissions during their declining phase. The time to the concentration peak increases with the ratio μ_4/μ_3 , describing the ratio of the impulse response of atmospheric CO_2 coming from the infinite time-constant τ_4 and finite but long time-constant τ_3 respectively. It increases with the time t_1 to the emissions peak, and in fact the influence of t_1 can be described in terms of dimensionless ratio t_1/τ_3 . However the left side of equation (13) increases with t_1 , because of the exponential term, so the influence of t_1 is not as strong as equation (14), which neglects the influence of this term, would suggest. Similarly the influence of τ_3 is not as strong as equation (14) would suggest because the exponential term in equation (13) also increases with this quantity.

We can write equation (13) in terms of dimensionless variables $m'_{av,i}/m'_{av,d}$, t_1/τ_3 , and parameter μ_4/μ_3 , by writing it in equivalent form

$$\tau_3 e^{-t_2/\tau_3} \left(\frac{m'_{av,i}}{m'_{av,d}} - \left(\frac{m'_{av,i}}{m'_{av,d}} - 1 \right) e^{t_1/\tau_3} \right) - \frac{\mu_4}{\mu_3} \frac{m'_d}{m'_{av,d}} t_2 \cong \tau_3 + \frac{\mu_4}{\mu_3} \left(\frac{m'_i}{m'_{av,d}} - \frac{m'_d}{m'_{av,d}} \right) t_1 \quad (15)$$

The above model depends also on variables $m'_i/m'_{av,d}$ and $m'_d/m'_{av,d}$. Recall that these are ratios of un-weighted rates of change and weighted rates of change, i.e. weighted by e^{t/τ_3} . It is shown later that $m'_d/m'_{av,d}$ is approximately constant across scenarios (also see Figure 3).

Figure 2 plots the time-delay between peak emissions and concentrations versus the dimensionless variable $m'_{av,i}/m'_{av,d}$, for the emissions scenarios plotted in Figure 1. The time-delay increases with the absolute value of this ratio. Furthermore, for scenarios with relatively short e-folding mitigation timescale, less than about 40 years, the relationship is approximately linear for the family of scenarios considered here.

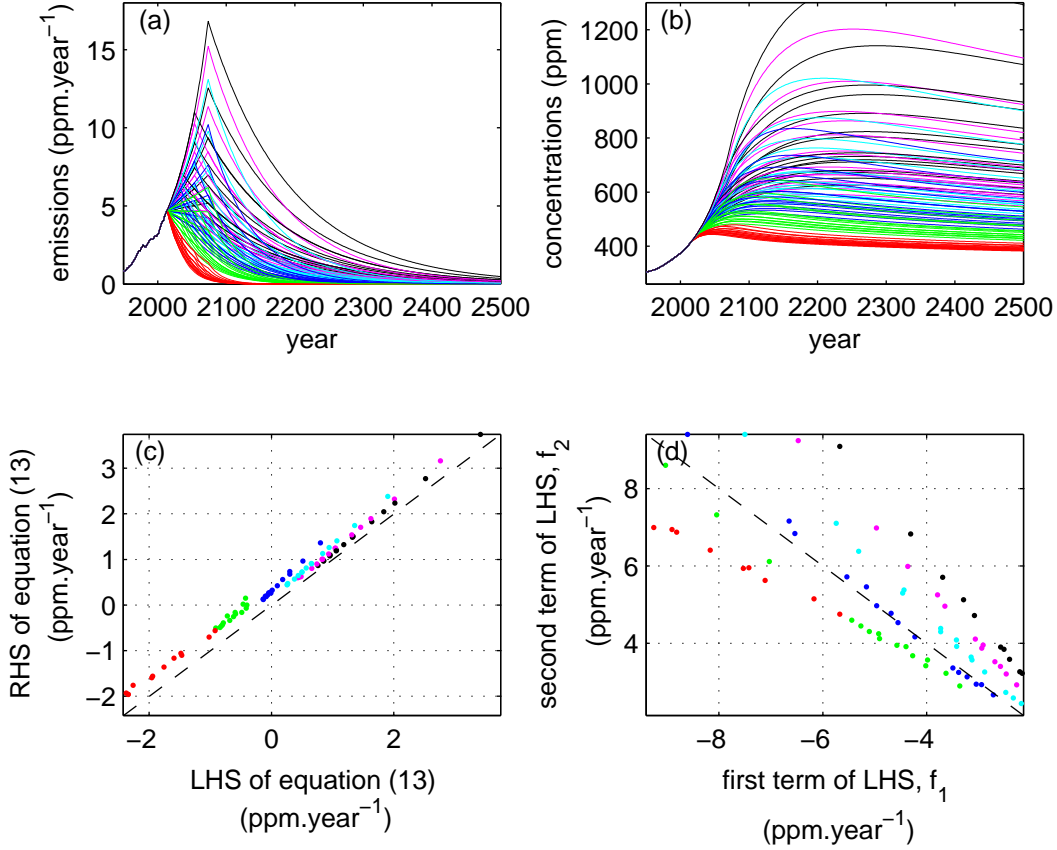


Figure 1: Verification of approximation in equation (13) and demonstration that neither term on the left side of this equation is dominant for a wide range of CO₂ emissions scenarios: (a) emissions pathways; (b) corresponding graphs of CO₂ concentration; (c) each side of equation (13), with left side in abscissa and right side in ordinate. The dashed line shows where abscissa and ordinate are equal; (d) first and second terms on the left side of equation (13), $f_1 = \mu_3 \tau_3 e^{-t_2/\tau_3} \left(m'_{av,i} - (m'_{av,i} - m'_{av,d}) e^{t_1/\tau_3} \right)$ and $f_2 = -\mu_4 m'_d t_2$, showing that neither term is dominant. Dashed line shows where abscissa and ordinate are of equal magnitude. Colors in panels correspond to different e-folding mitigation timescales in the respective emissions scenarios (see legend in Figure 2).

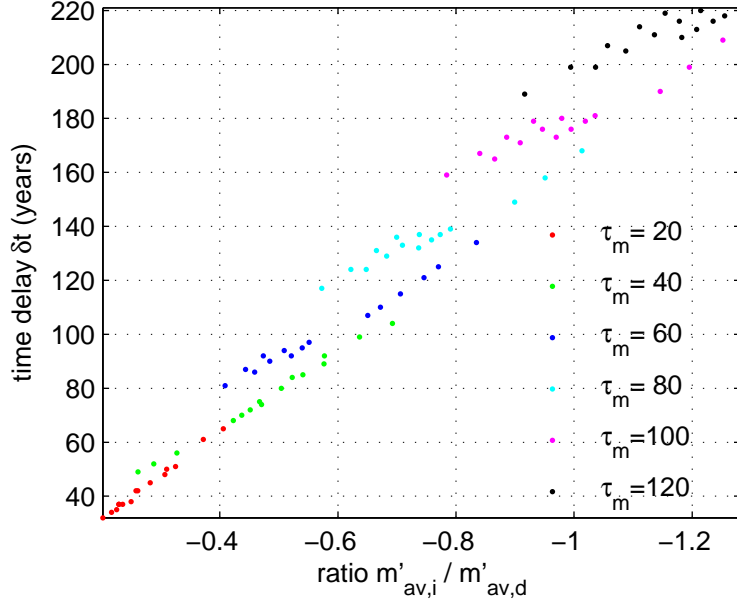


Figure 2: Time-delay δt between peak emissions and concentrations versus the ratio of the weighted-averaged rate of increase and decrease of emissions, defined in equations (6) and (7) respectively. The time delay increases with the absolute value of this ratio. Note the reversed axis in the abscissa. For short e-folding mitigation timescales, the relationship is approximately linear. Results are shown for the scenarios in Figure 1, and colors in the plot correspond to the colors in Figures 1a-d.

3.2 Influence of mitigation parameters, and importance of e-folding mitigation timescale

Here we consider the effects of parameters of our specific emissions model on the rates $m'_{av,i}$, $m'_{av,d}$, m'_i and m'_d , and thereby on the solution to equation (15). This discussion is of more general interest beyond this particular model because the parameters - the GGP growth rate, the e-folding mitigation timescale at which emissions intensity decreases, and the time to stabilization of GGP - can be easily interpreted. Considering first the rate of change of emissions during its increasing phase

$$m'_i = \frac{\int_0^{t_1} m'(z) dz}{\int_0^{t_1} dz} = \frac{m(t_1)}{t_1} \quad (16)$$

Approximating $(1+r)^t \cong e^{rt}$ the formula for emissions becomes $m(t_1) = m_0 e^{r(t_1-t_0)-(t_1-t_0)/\tau_m}$. If $r > 1/\tau_m$ then $t_1 = t_g$, i.e. emissions peaks when GGP is maximum. Otherwise $t_1 = t_0$, the present. In the first case $m(t_1) = m_0 e^{(r-\frac{1}{\tau_m})(t_g-t_0)}$ and $m'_i = m_0 e^{(r-\frac{1}{\tau_m})(t_g-t_0)}/t_g$, whereas in the second case $m(t_1) = m_0$ and $m'_i = m_0/t_0$.

In the following we consider only the case where $r > 1/\tau_m$ because global emissions of CO₂ are expected to

continue to increase for a while. For its decreasing phase

$$m'_d = \frac{\int_{t_1}^{t_2} m'(z) dz}{\int_{t_1}^{t_2} dz} = \frac{m(t_2) - m(t_1)}{t_2 - t_1} \quad (17)$$

Then $m(t_2) = m(t_1) e^{-\frac{t_2-t_1}{\tau_m}}$ so that $m'_d = -m(t_1) \left(1 - e^{-\frac{t_2-t_1}{\tau_m}}\right) / (t_2 - t_1)$, which becomes $-m_0 e^{(r-\frac{1}{\tau_m})(t_g-t_0)} \left(1 - e^{-\frac{t_2-t_g}{\tau_m}}\right)$.

For the weighted rate of increase of emissions between 0 and t_1 , we decompose the integral in the numerator of equation (6) into that between 0 and t_0 and between t_0 and t_1 . Using $m'(t) = \left(r - \frac{1}{\tau_m}\right) m_0 e^{(r-\frac{1}{\tau_m})(t-t_0)}$ between t_0 and t_1 we obtain

$$m'_{av,i} = \frac{m'_{av0} + m_0 \frac{r-\frac{1}{\tau_m}}{r-\frac{1}{\tau_m}+\frac{1}{\tau_3}} \left(e^{(r-\frac{1}{\tau_m})(t_g-t_0)+\frac{t_g}{\tau_3}} - e^{\frac{t_0}{\tau_3}} \right)}{\tau_3 (e^{t_g/\tau_3} - 1)} \quad (18)$$

where $m'_{av0} = \int_0^{t_0} e^{z/\tau_3} m'(z) dz$. Likewise using $m'(t) = -\frac{m_0}{\tau_m} e^{r(t_g-t_0)-\frac{t-t_0}{\tau_m}}$ between t_1 and t_2

$$m'_{av,d} = -\frac{m_0 e^{(r-\frac{1}{\tau_m})(t_g-t_0)} \left(e^{\frac{t_g}{\tau_3}} - e^{-\frac{t_2-t_g}{\tau_m}+\frac{t_2}{\tau_3}} \right)}{(\tau_3 - \tau_m) (e^{t_2/\tau_3} - e^{t_g/\tau_3})} \quad (19)$$

Let us compare the rates m'_d and $m'_{av,d}$ by considering ratio

$$\frac{m'_{av,d}}{m'_d} = \frac{e^{\frac{t_g}{\tau_3}} \left(1 - e^{-\frac{t_2-t_g}{\tau_m}+\frac{t_2-t_g}{\tau_3}} \right)}{1 - e^{-\frac{t_2-t_g}{\tau_m}}} \frac{t_2 - t_g}{(\tau_3 - \tau_m) e^{\frac{t_g}{\tau_3}} \left(e^{\frac{t_2-t_g}{\tau_3}} - 1 \right)} \quad (20)$$

which, assuming $1/\tau_m \gg 1/\tau_3$ because the mitigation timescale is generally much shorter than this time-constant of CO₂, simplifies to

$$\frac{t_2 - t_g}{(\tau_3 - \tau_m) \left(e^{\frac{t_2-t_g}{\tau_3}} - 1 \right)} \quad (21)$$

In case $\frac{t_2-t_g}{\tau_3} \ll 1$ the exponential term in equation (21) simplifies to $1 + \frac{t_2-t_g}{\tau_3}$, in which case the ratio approximates to

$$\frac{m'_{av,d}}{m'_d} \cong \frac{\tau_3}{\tau_3 - \tau_m} \quad (22)$$

which, for cases where $\tau_3 \gg \tau_m$, approximates to 1. Hence the value of weighted average $m'_{av,d}$, while not exactly equal that of the average rate of change of emissions m'_d in its decreasing phase, closely approximates the latter, especially in cases of rapid mitigation when, in addition, the time-delay between peak emissions

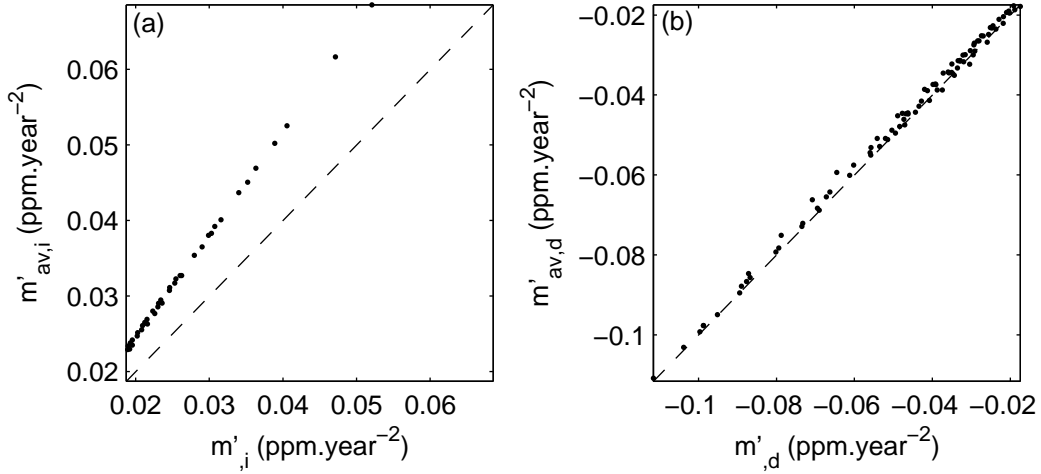


Figure 3: Plots of weighted (by e^{t/τ_3}) versus unweighted rate of change of emissions: (a) increasing phase; (b) decreasing phase. Dashed lines show where abscissa and ordinate are equal. An approximate linear relationship holds between the two sets of variables.

and concentrations is short compared to time-constant τ_3 . A near-equality between these two variables, weighted and unweighted, is seen in Figure 3b. Similarly Figure 3a shows the relationship between weighted and unweighted rates of change for the phase when emissions are increasing. While departure from equality is larger, the relationship is still approximately linear.

Although these weighted and unweighted rates are not the same, because of their similarities we can examine ratio m'_i/m'_d , which is more tractable, to understand qualitatively what controls the behavior of ratio $m'_{av,i}/m'_{av,d}$. The former ratio can be written as

$$\frac{m'_i}{m'_d} = -\frac{t_2 - t_g}{t_g} \frac{1}{1 - e^{-\frac{t_2 - t_g}{\tau_m}}} \quad (23)$$

A longer time t_2 to the concentration peak by itself can increase the above ratio, because the graph of emissions is convex in its decreasing phase, so that its slope is decreasing. The influence of time t_g to peak emissions is weak because of two countervailing influences: shorter t_g increases the magnitude of the numerator as well as that of the denominator above. The main influence on this ratio is that of mitigation timescale τ_m . Short τ_m decreases the magnitude of this ratio.

The strong influence of the mitigation timescale on the ratio $m'_{av,i}/m'_{av,d}$ is seen in Figure 4. Short mitigation timescale, corresponding to rapid mitigation of emissions intensity, is therefore essential to limit the absolute magnitude of this ratio and therefore assure a short time-delay.

Note that the ratio in equation (23) does not depend on the GGP's growth rate. While this rate influences

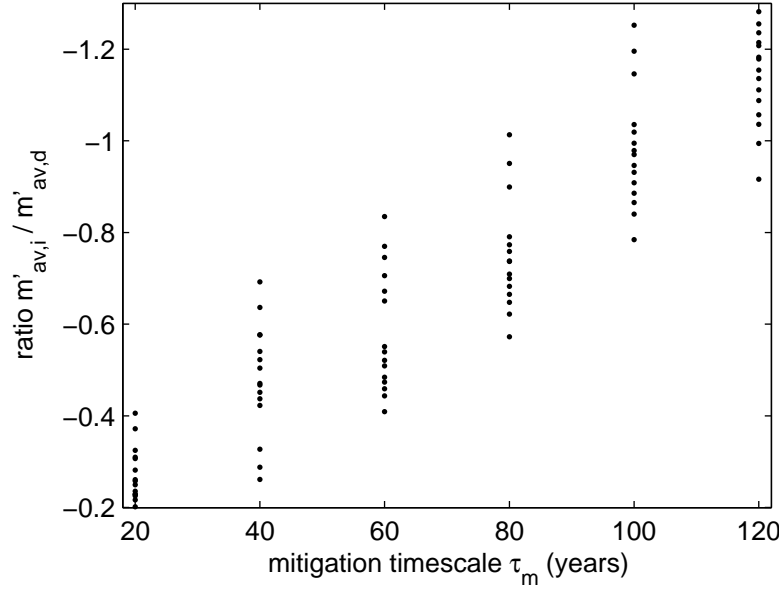


Figure 4: Ratio $m'_{av,i}/m'_{av,d}$ versus e-folding mitigation timescale. Note the reversed axis on the ordinate. Short mitigation timescale decreases the absolute value of this ratio.

peak emissions of CO_2 , it affects the average growth rate and decrease of emissions in the same manner and hence is not a factor in this ratio and consequently in the time-delay between peak emissions and concentrations.

3.3 Carbon cycle uncertainties

As indicated earlier the carbon-cycle parameters affecting the time to the concentration peak are the multi-century time-constant τ_3 and the ratio μ_4/μ_3 , describing the ratio of the impulse response of atmospheric CO_2 from the infinite time-constant τ_4 and the long time-constant τ_3 respectively. Figure 5 plots the influence of these parameters on the time-delay. These are the uncertainties in the carbon cycle that must be constrained in order to constrain forecasts of the timing of peak concentrations of CO_2 .

4 Conclusions and Discussion

The results presented here are based on approximating the linear carbon cycle model with four time-constants of *Joos et al.* (2013), with one time-constant being infinite because a fraction of emitted CO_2 persists for a very long time (*Archer* (2005); *Archer and Brovkin* (2008)). We identified the main factors governing the time-delay between peak CO_2 emissions and concentrations.

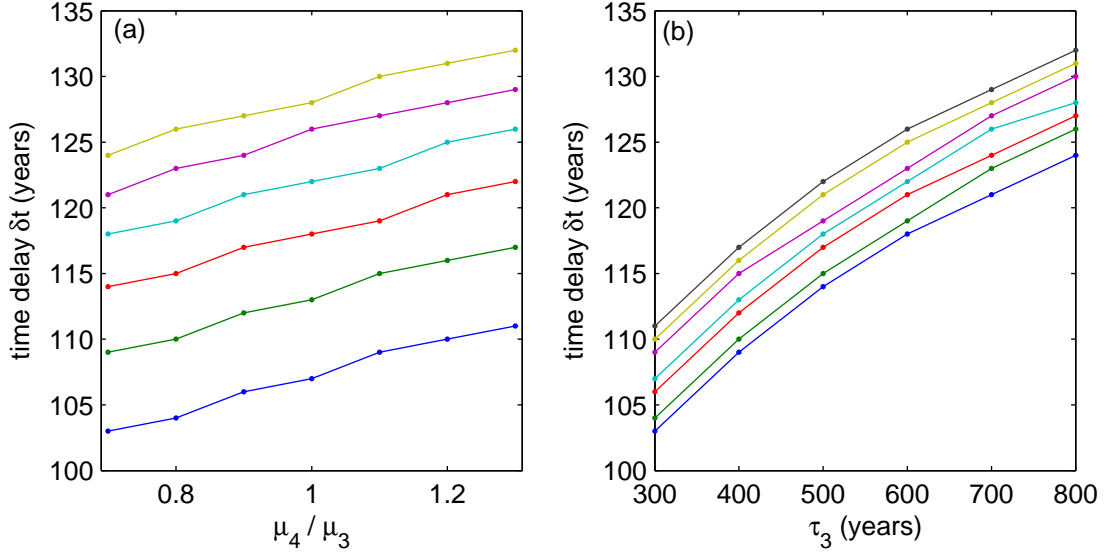


Figure 5: The main carbon cycle parameters affecting the time-delay between peak CO₂ emissions and concentrations: (a) relation between time-delay and μ_4/μ_3 describing the ratio of the impulse response of atmospheric CO₂ from the infinite time-constant τ_4 and the long time-constant τ_3 respectively; (b) relation between time-delay and long time-constant τ_3 . In each panel, different curves correspond to different fixed values of the other parameter. The emissions scenario used has $t_g - t_0 = 60$ years, $r = 0.015$ %/year, and $\tau_m = 50$ years.

On the emissions front, the main factor is the e-folding timescale with which the mitigation of emissions intensity of GGP ("decarbonization") occurs. This can be viewed as the inverse of the corresponding mitigation rate (*Seshadri (2015a)*). Short decarbonization timescale leads to short time-delay between emissions and concentration peaks. Therefore achieving decarbonization rapidly is important to achieving an early peak in CO₂ concentrations.

The time-delay between peak emissions and concentrations is not sensitive to the time to peak emissions. However an early emissions peak will facilitate an early concentration peak.

The growth rate of economic output is an important factor in peak emissions. However as discussed here it does not affect the time delay between peak emissions and concentrations, because it has the same effect on the rate of increase of emissions and the rate of decrease of emissions, whose ratio governs the time-delay. Therefore in this model where peak emissions corresponds to where GGP stabilizes, the timing of peak concentrations is not affected by this growth rate. Nevertheless it is obviously important in considering the magnitude of peak emissions, and faster economic growth has to be accompanied by more rapid decarbonization.

The important non-dimensional parameter in our discussion has been the ratio of the rate of increase of emissions and the rate of decrease of emissions. Limiting the magnitude of this ratio will help achieve an

early peak, and this can be accomplished by keeping the mitigation timescale short.

With respect to the atmospheric cycle of CO₂, the influential parameters are the long but finite time-constant τ_3 that occurs on century-scales, and the factor μ_4/μ_3 describing the ratio of the impulse response function of atmospheric CO₂ from the infinite time-constant τ_4 and the long time-constant τ_3 respectively. The time-delay increases with these parameters. The short time constants τ_1 and τ_2 occurring on decadal scales or less play a small role in the long-term dynamics of atmospheric CO₂, and uncertainties in their values are correspondingly less important for forecasting the concentration peak.

In summary it is important to constrain these carbon cycle parameters, in addition to achieving an early mitigation peak, as well as implementing decarbonization on short timescales.

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